

Production of a Heavy Quarkonium with a Photon or via ISR at Z Peak in e^+e^- Collider

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Abstract

Considering the possibility to build an e^+e^- collider at the energies around Z -boson resonance with a luminosity so high as $\mathcal{L} \propto 10^{34} \text{cm}^{-2} \text{s}^{-1}$ (even higher) and the abilities of a modern synthesis detector, we systematically calculate the exclusive two body processes: e^+e^- annihilates into a heavy quarkonium and a photon (initiate state radiation i.e. ISR is involved), at the energies around the Z -boson resonance. Since the couplings of Z -boson to quarks contain axial vector as well as vector, so here the produced heavy quarkonium may stand for a charmonium such as $J/\psi, \eta_c, h_c, \chi_{cJ} \cdots$ and a bottomonium such as $\Upsilon, \eta_b, h_b, \chi_{bJ} \cdots$ respectively. If we call such a collider with so high luminosity and running around the Z -boson resonance as a Z -factory, then our results obtained here indicate that experimental studies at a Z -factory about the various heavy quarkona (their ground and excited states) via the two-body processes, especially, the production of the bound states with quantum number $J^{PC} = 1^{--}$ via ISR, have outstanding advantages.

Key Words: Heavy Quarkonium, Exclusive Two-body Production, Z -factory

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The production of a heavy quarkonium, such as a charmonium or a bottomonium, at high energies not only may be used to test the high-energy behavior of quantum chromodynamics (QCD) as well as the interplay of perturbative and non-perturbative phenomena in QCD, but also may be used to search for suitable sources to produce experimental example for studying the properties of the heavy quarkonia. Comparing with the hadronic colliders such as TEVATRON and LHC, an e^+e^- collider has many advantages except the low rate of the production in studying the production and/or in using the produced heavy quarkonium as an experimental study sample. Recently the technique progress on e^+e^- colliders motivated by ILC has achieved, that the luminosity may be raised so high as $\mathcal{L} \propto 10^{34} cm^{-2}s^{-1}$ (even higher). Moreover, if such an e^+e^- collider further runs at Z -boson peak energy, the resonance effects at the peak may raise the production rate to several magnitude order high, thus the shortcoming (the low production rate) can be overcome much for the mentioned purposes. For convenience, we will call as a collider as a ‘ Z -factory’ later on¹. As estimated in Ref.[1], in terms of GigaZ, one can perform experiments on the basis of an example more than $10^{9\sim 10}$ Z -events, that enlarges the number of Z -events by $3 \sim 4$ orders of magnitude than that collected in LEP-I. Thus we think Z -factory will be able to open new opportunities not only for high precision physics in the electro-weak sector, but also for the hadron physics including that of the heavy quarkonia.

Considering the abilities of a modern synthesis detector for photons, and the fact that the energy-momentum is monochromatic in a two-body process, who’s final state contains two body only and incoming energy is fixed, in the present paper we would like to focus the two body processes for the heavy quarkonium production as below:

$$e^+(p_1) + e^-(p_2) \rightarrow \gamma(p_3) + H_{Q\bar{Q}}(P) \quad (1)$$

where the heavy quarkonium state $H_{Q\bar{Q}}$ stands for J/ψ , η_c , h_c , χ_{cJ} ($J = 0, 1, 2$) for charmonium and stands for Υ , η_b , h_b , χ_{bJ} ($J = 0, 1, 2$) for bottomonium respectively², especially at the energy of Z -peak. Since the produced heavy quarkonium in the two-body processes is monochromatic in energy-momentum, so experimentally the heavy quarkonium may be

¹ In the literature (Ref.[1] and references therein) running the prospective high-energy e^+e^- collider is called as GigaZ.

² In fact, here $H_{Q\bar{Q}}$ can stand for higher excited states too, but in the present paper we restrict ourselves to consider the low-lying excited states only.

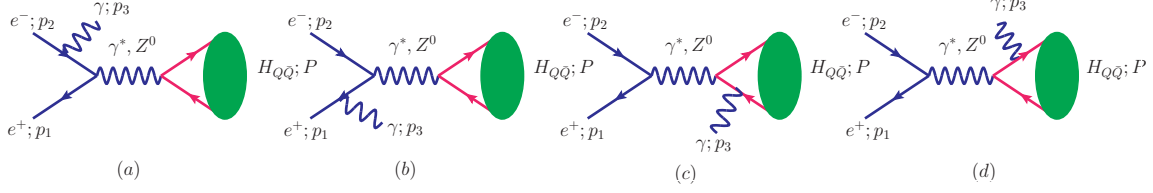


FIG. 1: Typical Feynman diagrams for the process $e^+(p_1) + e^-(p_2) \rightarrow \gamma^*/Z^0 \rightarrow \gamma(p_3) + H_{Q\bar{Q}}(P)$, where $H_{Q\bar{Q}}$ equals to J/ψ , η_c , χ_{cJ} ($J = 0, 1, 2$) for charmonium and Υ , η_b , χ_{bJ} ($J = 0, 1, 2$) for bottomonium respectively.

comparatively easy to be identified, as long as the investigated event-sample is big enough. Thus how great the cross sections are crucial for experimental studies of the heavy quarkonia. Moreover to observe the production of the heavy quarkonia at Z -peak via the two body process, the resonance effects i.e. the contributions from ‘ Z -boson exchange’ become greater than those from ‘virtual-photon exchange’ owing to the Z -boson propagator comes to its mass-shell, and ‘ Z -boson exchange’ is of weak interaction (without parity and charge-parity conservation), hence various states of quarkonia with different parity and charge-parity can be produced quite equally. To the leading order calculation, there are four typical Feynman diagrams FIG.1a, FIG.1b, FIG.1c and FIG.1d for the processes (since each typical Feynman diagram in FIG.1 contains two diagrams: a virtual-photon exchange one and a virtual Z -boson exchange one, so the total number of the independent Feynman diagrams represented in FIG.1 is eight in fact) if the the heavy quarkonia in the final state have quantum numbers as $J^{PC} = 1^{--}$, whereas there are two FIG.1c and FIG.1d only (for the same reason as in the case for the heavy quarkonia $J^{PC} = 1^{--}$, the number of independent Feynman diagrams are four) if the the heavy quarkonia in the final state have quantum numbers different from $J^{PC} = 1^{--}$. As matter of fact, FIG.1a and FIG.1b are those of ISR (here one may see that ISR produce the states with quantum numbers $J^{PC} = 1^{--}$ only). Note here that due to the fact that a heavy-quark propagator via two vertices connects with the heavy quarkonium as shown in the diagrams FIG.1c and FIG.1d, but not as that shown in the diagrams FIG.1a and FIG.1b, a photon directly connects to the heavy quarkonium via a single vertex, then the heavy quarkonium, involving orbital angle-momentum excitation ($L > 0$), can be also produced via this mechanism depicted by the diagrams FIG.1c and FIG.1d.

The calculations on the processes, no matter for the ground state or for the excited states,

need to establish the exact relations between each term of the amplitude (each Feynman diagram) and the nonperturbative matrix elements in NRQCD framework[2], or equivalently, the relevant Bethe-Salpeter (B.S.) wave function (derivative of the wave function) at origin for the S -(P -)wave production respectively in B.S framework[3].

Generally, according to Feynman rules, the term of the amplitude relating to the concerned Feynman diagram, can be written as

$$M_k = \mathcal{C} \bar{v}_e(p_1) \Gamma_{1k} u_e(p_2) i \int \frac{d^4 q}{(2\pi)^4} \text{Tr} [\bar{\chi}(P, q) \Gamma_{2k}] , \quad (2)$$

where $\bar{\chi}(P, q)$ is the B.S wave function for the concerned quarkonium, \mathcal{C} stands for an overall parameter, which is the same for all the Feynman diagrams. Γ_{1k} stands for the structure of the k -th Feynman diagram, and is for the electric line, which includes the relative string of Dirac γ matrices and the corresponding scalar part of the propagators, Γ_{2k} stands for the similar structure of the heavy quark line. For convenience, the intermediate photon or Z propagator is put into Γ_{1k} .

As for a quarkonium in S -wave state, in the non-relativistic approximation, $\bar{\chi}(P, q)$ can be written as

$$\bar{\chi}(P, q) = \phi(q) \frac{1}{2\sqrt{M}} (\alpha \gamma_5 + \beta \hat{e}(S_z)) (\hat{P} + M) , \quad (3)$$

where P , $M \simeq m_Q + m_{\bar{Q}}$ are the momentum, the mass of the heavy quarkonium respectively and q is the relative momentum between the two constituent quarks. In this paper, we use \hat{a} to denote the contraction between the Dirac γ matrix and a momentum or polarization vector a , i.e. we use \hat{a} instead of \not{a} . $\alpha = 1$, $\beta = 0$ for the pseudoscalar ($[^1S_0]$) quarkonium and $\alpha = 0$, $\beta = 1$ for the vector ($[^3S_1]$) one. The wave function $\phi(q)$, the radial part of the momentum space, can be related to the space-time wave function at origin by the integration, $i \int \frac{d^4 q}{(2\pi)^4} \phi(q) = \psi(0)$, where $|\psi(0)|^2 = |R(0)|^2/(4\pi)$. Then the S -wave amplitude can be simplified as

$$M_k = \mathcal{C} \bar{v}_e(p_1) \Gamma_{1k} u_e(p_2) \psi(0) \text{Tr} \left[\frac{1}{2\sqrt{M}} (\alpha \gamma_5 + \beta \hat{e}(S_z)) (\hat{P} + M) \Gamma_{2k}^0 \right] , \quad (4)$$

where Γ_{2k}^0 is defined by the following Eq.(9).

As for a quarkonium in P -wave state, in the non-relativistic approximation, the B.S. wave function $\bar{\chi}(P, q) = \bar{\chi}^{[2S+1]P_J J_z}(P, q)$ can be written as

$$\begin{aligned} \bar{\chi}^{[2S+1]P_J J_z}(P, q) &\simeq \sum_{S_z, \lambda, \lambda'} \frac{-\sqrt{M}}{4m_Q m_{\bar{Q}}} \cdot \Psi(q) \cdot (\epsilon^\lambda \cdot q) \cdot \\ &(\hat{q}_{\bar{Q}} - m_{\bar{Q}}) \cdot (\delta_{S,0} \delta_{S_z,0} \gamma_5 + \delta_{S,1} \delta_{S_z, \lambda'} \hat{\epsilon}^{\lambda'}) \cdot (\hat{q}_Q + m_Q) \cdot \langle 1\lambda; S S_z | J J_z \rangle, \end{aligned} \quad (5)$$

where q_Q and $q_{\bar{Q}}$ are the momenta of Q and \bar{Q} quarks inside the quarkonium:

$$q_{\bar{Q}} = \alpha_1 P - q, \quad q_Q = \alpha_2 P + q, \quad \alpha_1 = \frac{m_{\bar{Q}}}{m_Q + m_{\bar{Q}}}, \quad \alpha_2 = \frac{m_Q}{m_Q + m_{\bar{Q}}}. \quad (6)$$

For convenience, we introduce $q_{\perp}^{\mu} \equiv q^{\mu} - q_{\parallel}^{\mu}$, $q_{\parallel}^{\mu} \equiv \frac{(P \cdot q)}{M^2} P^{\mu}$ and $q_{\parallel} \equiv |q_{\parallel}^{\mu}|$, hence accordingly we have $d^4 q = dq_{\parallel} d^3 q_{\perp}$. $\Psi(q)$ stands for the ‘ P -wave scalar wave function’, and $\langle 1\lambda; SS_z | JJ_z \rangle$ is the Clebsch-Gordon coefficient for L-S coupling. The spin structure of the B.S. wave function $\bar{\chi}^{[(2S+1)P_J, J_z]}(q)$ defined in Eq.(5) is of the lowest order in q for the P -wave state production.

The derivative of the wave function at origin $\psi'_0(0)$ in coordinate representation relates to the P -wave scalar function $\Psi(q)$ in Eq.(5) under the so-called instantaneous approximation by the integration:

$$i \int \frac{dq_{\parallel} d^3 q_{\perp}}{(2\pi)^4} q^{\alpha} \Psi(q) (\epsilon^{\lambda} \cdot q) = i \int \frac{d^3 q_{\perp}}{(2\pi)^3} \tilde{q}_{\perp}^{\alpha} \phi\left(-\frac{q_{\perp}^2}{M^2}\right) (\epsilon^{\lambda} \cdot q_{\perp}) = i \epsilon^{\lambda \alpha} \psi'_0(0), \quad (7)$$

where $|\psi'_0(0)|^2 = |R'(0)|^2/(4\pi)$. Here \tilde{q}_{\perp} is the unit vector $\frac{q_{\perp}}{\sqrt{-q_{\perp}^2}}$, and $\phi(-\frac{q_{\perp}^2}{M^2})$ stands for the P -wave ‘scalar wave function’ in the sense of potential model. Substituting Eq.(6) into Eq.(5), we obtain

$$\begin{aligned} i \int \frac{dq_{\parallel}}{(2\pi)} \bar{\chi}^{[(2S+1)P_J, J_z]}(P, q) &\simeq \sum_{S_z, \lambda, \lambda'} \phi\left(-\frac{q_{\perp}^2}{M^2}\right) (\epsilon^{\lambda} \cdot q_{\perp}) \langle 1\lambda; SS_z | JJ_z \rangle \\ &\cdot \left\{ \frac{1}{2\sqrt{M}} (-\hat{P} + M) \cdot (\delta_{S,0} \delta_{S_z,0} \gamma_5 + \delta_{S,1} \delta_{S_z,\lambda'} \hat{\epsilon}^{\lambda'}) \right. \\ &- \left(\frac{\sqrt{M}}{4m_Q m_{\bar{Q}}} \right) \cdot \left[\alpha_2 \hat{q}_{\perp} (-\hat{P} + M) (\delta_{S,0} \delta_{S_z,0} \gamma_5 + \delta_{S,1} \delta_{S_z,\lambda'} \hat{\epsilon}^{\lambda'}) \right. \\ &\left. \left. + \alpha_1 (-\hat{P} + M) \cdot (\delta_{S,0} \delta_{S_z,0} \gamma_5 + \delta_{S,1} \delta_{S_z,\lambda'} \hat{\epsilon}^{\lambda'}) \hat{q}_{\perp} \right] + \mathcal{O}(q_{\perp}^2) \right\} \\ &= \sum_{S_z, \lambda, \lambda'} \phi\left(-\frac{q_{\perp}^2}{M^2}\right) (\epsilon^{\lambda} \cdot q_{\perp}) \langle 1\lambda; SS_z | JJ_z \rangle (A_{SS_z}^{\lambda'} + B_{SS_z}^{\lambda' \mu} q_{\perp \mu} \\ &\quad + \mathcal{O}(q_{\perp}^2)), \end{aligned} \quad (8)$$

where

$$\begin{aligned} A_{SS_z}^{\lambda'} &\equiv \frac{1}{2\sqrt{M}} (-\hat{P} + M) \cdot (\delta_{S,0} \delta_{\lambda',0} \gamma_5 + \delta_{S,1} \delta_{S_z,\lambda'} \hat{\epsilon}^{\lambda'}), \\ B_{SS_z}^{\lambda' \mu} &\equiv - \left(\frac{\sqrt{M}}{4m_Q m_{\bar{Q}}} \right) \cdot \left[\alpha_2 \gamma^{\mu} (-\hat{P} + M) \cdot (\delta_{S,0} \delta_{\lambda',0} \gamma_5 + \delta_{S,1} \delta_{S_z,\lambda'} \hat{\epsilon}^{\lambda'}) \right. \\ &\quad \left. + \alpha_1 (-\hat{P} + M) \cdot (\delta_{S,0} \delta_{\lambda',0} \gamma_5 + \delta_{S,1} \delta_{S_z,\lambda'} \hat{\epsilon}^{\lambda'}) \gamma^{\mu} \right]. \end{aligned}$$

TABLE I: Functions $C^{(2S+1)P_J}$, $E_\mu^{[2S+1P_J]J_z}$ and $F^{[2S+1P_J]J_z}$ for a particular P -wave heavy quarkonium states, where ϵ_μ^λ and $\epsilon_{\mu\nu}^{J_z}$ are polarization vector and tensor respectively.

P -wave state	$E_\mu^{[2S+1P_J]J_z}$	$F^{[2S+1P_J]J_z}$	$C^{(2S+1)P_J}$
1P_1	$(-\hat{P} + M)\gamma_5\epsilon_\mu^\lambda$	$-\frac{\hat{P}\hat{\epsilon}^\lambda\gamma_5 + M(\alpha_1 - \alpha_2)\hat{\epsilon}^\lambda\gamma_5}{2M\alpha_1\alpha_2}$	$\frac{1}{2\sqrt{M\sqrt{N_c}}}$
3P_0	$(-\hat{P} + M)(M\gamma_\mu + P_\mu)$	$-\frac{3[(\alpha_2 - \alpha_1)\hat{P} + M]}{2\alpha_1\alpha_2}$	$\frac{1}{2\sqrt{3N_c}M^{3/2}}$
3P_1	$i(\hat{P} + M)\epsilon_\mu^{\rho\varrho\nu}\gamma_\rho P_\varrho\epsilon_\nu^\lambda$	$-\frac{\hat{P}\hat{\epsilon}^\lambda\gamma_5 - M\hat{\epsilon}^\lambda\gamma_5}{\alpha_1\alpha_2}$	$\frac{1}{2\sqrt{2N_c}M^{3/2}}$
3P_2	$(-\hat{P} + M)\gamma_5\gamma_\nu\epsilon_{\mu\nu}^{J_z}$	0	$\frac{1}{2\sqrt{N_c}M}$

To leading order of the relativistic approximation and according to Eq.(2), the next step is to do the expansion of the Γ_{2k} about q_μ up to $\mathcal{O}(q^2)$ for the P -wave production, i.e.

$$\Gamma_{2k} = \Gamma_{2k}^0 + \Gamma_{2k}^\mu \cdot q_\mu + \mathcal{O}(q^2), \quad (9)$$

where Γ_{2k}^0 and Γ_{2k}^μ are independent of q_μ . Substituting the above equations into Eq.(2) and carrying out the integration over $d^4q = dq_\parallel d^3q_\perp$ and with the help of Eq.(7), we obtain

$$M_k = \mathcal{C}\bar{v}_e(p_1)\Gamma_{1k}u_e(p_2)\sum_{S_z,\lambda,\lambda'}\langle 1\lambda; SS_z|JJ_z\rangle\psi'(0)\epsilon_\mu^\lambda \cdot Tr\left[A_{SS_z}^{\lambda'} \cdot \Gamma_{2k}^\mu + B_{SS_z}^{\lambda'\mu} \cdot \Gamma_{2k}^0\right]. \quad (10)$$

For a specific P -wave state, summing over the explicit λ and λ' for the Clebsch-Gordon coefficients, Eq.(10) can be further simplified as

$$M_k^{S,JJ_z} = \mathcal{C}\bar{v}_e(p_1)\Gamma_{1k}u_e(p_2)C^{2S+1P_J}\psi'(0)Tr\left[E_\mu^{[2S+1P_J]J_z} \cdot \Gamma_{2k}^\mu + F^{[2S+1P_J]J_z} \cdot \Gamma_{2k}^0\right], \quad (11)$$

where the overall factor $C^{(2S+1)P_J}$ and the functions $E_\mu^{[2S+1P_J]J_z}$ and $F^{[2S+1P_J]J_z}$ are shown in TAB.I.

To do the computation we adopt the ‘linear polarization’ for the heavy quarkonium vector and tensor states accordingly, and their explicit formulation can be found in Ref.[4]. The amplitude of all mentioned processes can be easily generated by using FDC³. To simplify the amplitude, firstly we establish a complete set of ‘basic spinor lines’, such as that they are constructed by the multiplication of Dirac γ matrixes in a certain way as $\hat{p}_1\hat{p}_2\hat{p}_3\cdots$, then we consider them as bases to expand every terms of the amplitude and sum up all the

³ The relevant Fortran program for the mentioned processes is available upon request.

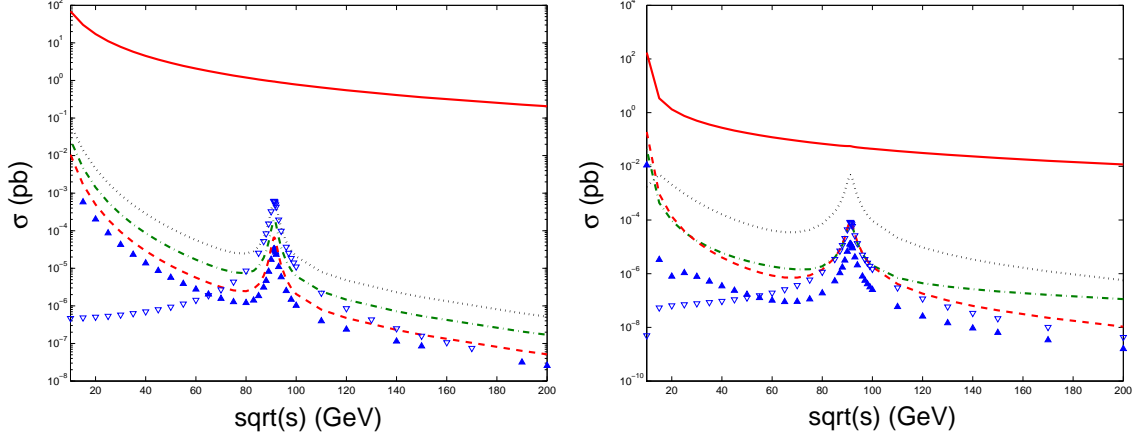


FIG. 2: (color online) Total cross sections for the processes $e^- + e^+ \rightarrow \gamma + H_{Q\bar{Q}}$ versus the collision energy. The red solid, the black dotted, the blue up-solid-triangle, the green dash-dotted, the red dashed and the down-hollow-triangle lines stand for $Q\bar{Q}$ in 3S_1 , 1S_0 , 3P_0 , 3P_1 , 3P_2 , 1P_1 respectively. The left figure is for charmonium and the right one is for bottomonium.

terms according to the expansion (the coefficients of the ‘bases’ are summed respectively). By this way, the amplitude and its square become quite condense, thus the efficiency for computing the amplitude squared is raised greatly. This kind of treatments can be carry out quite automatically in FDC package now[5].

When doing numerical calculations, we adopt

$$m_Z = 91.187 GeV, \quad m_b = 4.9 GeV, \quad m_c = 1.5 GeV, \quad (12)$$

and

$$\begin{aligned} |R_{1S}^{(c\bar{c})}(0)|^2 &= 0.810 GeV^3, \quad |R_{1S}^{(b\bar{b})}(0)|^2 = 6.447 GeV^3, \\ |R_{2P}^{(c\bar{c})}(0)|^2 &= 0.075 GeV^5, \quad |R_{2P}^{(b\bar{b})}(0)|^2 = 1.417 GeV^5 \end{aligned} \quad (13)$$

for the parameters of heavy quarkonia [6]. In order to have some idea about more samples to produce a ‘particle’ with such quantum numbers as $J^{PC} = 1^{--}$ via the ISR in addition to the ground states J/ψ and Υ ,

To present the results, the dependence of the total cross sections for the processes $e^- + e^+ \rightarrow \gamma + H_{Q\bar{Q}}$ with $H_{Q\bar{Q}}$ for charmonium and bottomonium on its collision energy is drawn in FIG.2, and the dependence of the differential cross sections for the processes $e^- + e^+ \rightarrow \gamma + H_{Q\bar{Q}}$ on $\cos \alpha$, where α stands for the angle between the quarkonium and

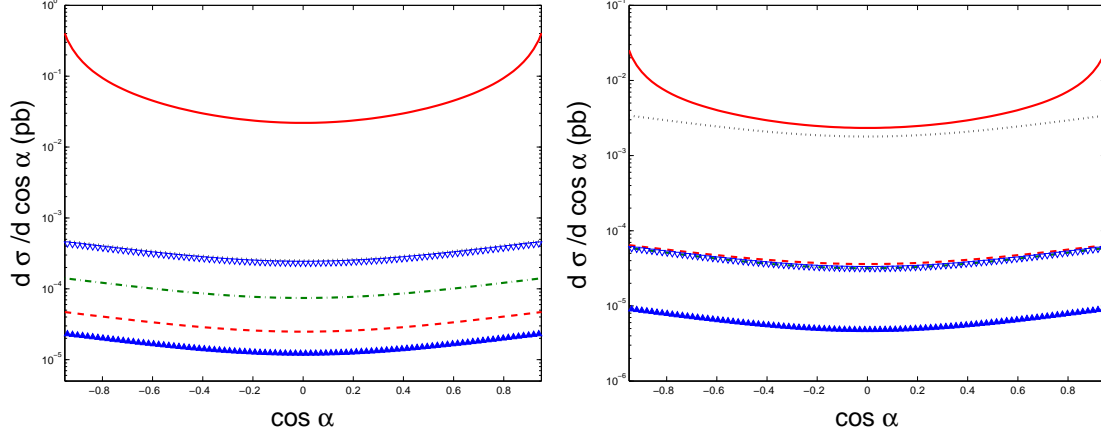


FIG. 3: (color online) Differential cross sections for the processes $e^- + e^+ \rightarrow \gamma + H_{Q\bar{Q}}$ versus $\cos \alpha$ at a C.M.S. energy as Z -mass. The red solid, the black dotted, the blue up-solid-triangle, the green dash-dotted, the red dashed and the blue down-hollow-triangle lines stand for $Q\bar{Q}$ in 3S_1 , 1S_0 , 3P_0 , 3P_1 , 3P_2 , 1P_1 respectively. The left figure is for charmonium (the dotted line and the blue down-hollow-triangle almost emerge together almost) and the right one is for bottomonium (the red dashed line, the green dash-dotted line and the blue down-hollow-triangle emerge together almost).

TABLE II: Total cross sections for the different quarkonium states at the collision energy $\sqrt{s} = m_Z$.

	3S_1	1S_0	3P_0	3P_1	3P_2	1P_1
$\sigma_{(c\bar{c})}(pb)$	0.934	0.662×10^{-3}	0.328×10^{-4}	0.197×10^{-3}	0.661×10^{-4}	0.615×10^{-3}
$\sigma_{(b\bar{b})}(pb)$	0.565×10^{-1}	0.475×10^{-2}	0.128×10^{-4}	0.838×10^{-4}	0.930×10^{-4}	0.833×10^{-4}

the beam, is drawn in FIG.3. From FIG.2, one can see clearly that as expected there is a peak at $\sqrt{s} = m_Z$ for the quarkonium states of 1S_0 , 1P_1 and 3P_J etc. It is because that in these processes the s -channel diagrams of Z -boson exchange FIG.1c and FIG.1d become dominant, when the Z -propagator in the diagrams approaches to mass-shell. We should emphasize that the Z -boson coupling is not only of vector, but also of axial vector, so the states with various quantum numbers, as long as they are lighter than m_Z , may be produced in terms of the s -channel diagrams of Z -boson exchange, and there must be a peak at $\sqrt{s} = m_Z$. In contrary, if the production is not at Z -mass energy but at an energy much

low, then of the s -channel diagrams FIG.1c and FIG.1d, Z -boson exchange ones become ignorable and only the ones of γ exchange play a role, so that there is a vector coupling only, and quite a lot heavy quarkonium states without suitable quantum numbers cannot be produced in terms of the ‘ s -channel mechanism’.

However, for a hidden flavored ‘comparatively light’ quarkonium ($m_{H_{Q\bar{Q}}} \ll m_Z$), when the quarkonium is a 3S_1 or 3D_1 state i.e. that with quantum numbers as $J^{PC} = 1^{--}$, then the t -channel photon-exchange diagrams, as shown in FIG.1a and FIG.1b, will be dominant over the other diagrams (including FIG.1c and FIG.1d even the Z -propagator being on-shell and the t -channel Z -boson-exchange ones in FIG.1a and FIG.1b). In fact, the ‘dominance’ is also the nature of ISR essentially. Thus the production of a $J^{PC} = 1^{--}$ heavy quarkonium is very large and without such a peak behavior around the energy of Z -boson as emerged in the production of some other states. To show the fact, the dependence of the cross-sections for producing this kind of quarkonia is also drawn in FIG.2.

Moreover, to make comparison, we put the total cross-sections for the production of the all low-lying states at Z -factory in TAB.II. We would like to note here that if varying $\sqrt{s} = m_Z \pm 0.5$ GeV slightly, the results will bring $\pm 1\%$ difference in the total cross sections.

From FIG.3, one may see that the distributions for the production of the heavy quarkonia are quite flat in directions, although in the forward and backward directions they arise in certain aspect, thus there is not essential loss on observing the heavy quarkonia, if putting a necessary cut around the beam directions.

To see the role quantitatively that ISR may play in the two-body final state processes at Z -factory, we assume a ‘ $X(4260)$ -like’ particle with quantum numbers $J^{PC} = 1^{--}$ and calculate its production precisely. For convenience, we denote the ‘ $X(4260)$ -like’ particle as $X_{b\bar{b}}$ below, and for definiteness we further assume its mass is $m_{X_{b\bar{b}}} \simeq 15$ GeV ($m_{X_{b\bar{b}}} > m_\Upsilon$), its decay constant is one third of that of Υ : $f_{X_{b\bar{b}}} = \frac{1}{3}f_\Upsilon$ and its main decay mode is similar to that of the particle $X(4260)$ [7], i.e., $X_{b\bar{b}} \rightarrow \Upsilon\pi\pi$, but not $X_{b\bar{b}} \rightarrow J/\psi\pi\pi$ so same as $X(4260)$. Finally we obtain the total cross-section of producing $X_{b\bar{b}}$ at Z_0 peak is $1.898 \cdot 10^{-3} pb$; while as a comparison, the total cross-section of producing Υ is $5.650 \cdot 10^{-2} pb$.

Summary: From the results obtained here, one may see that the production of a particle with Quantum numbers $J^{PC} = 1^{--}$ via the two-body final state processes $e^+ + e^- \rightarrow \gamma + H_{Q\bar{Q}}$ concerned here is of ISR essentially and the total cross sections for the production of the heavy quarkonium states at Z -boson peak via the two-body final state processes can be so

great $\sigma \simeq 1 \cdot 10^{-1} pb$ in magnitude, when $H_{Q\bar{Q}}$ is a heavy quarkonium with quantum numbers $J^{PC} = 1^{--}$; so great $\sigma \simeq 1 \cdot 10^{-2} pb$, when $H_{Q\bar{Q}}$ is a ‘X(4260)-like’ particle with quantum numbers $J^{PC} = 1^{--}$ as assumed in the above; whereas the total cross sections at Z -boson peak for producing the quarkonium states $H_{Q\bar{Q}}$ with the other quantum numbers (not $J^{PC} = 1^{--}$) via the two-body final state processes (without ISR) can only be $\sigma \simeq 10^{-3} \sim 10^{-4} pb$ in magnitude. Therefore, one can conclude that numerous events of $H_{Q\bar{Q}}$ with $J^{PC} = 1^{--}$ (the heavy quarkonia and even a ‘X(4260)-like particle such as $X_{b\bar{b}}$ as well) may be produced at a Z -factory. Also at such a Z -factory, hundreds, even thousands events of the heavy quarkonia $H_{Q\bar{Q}}$ with different quantum numbers from $J^{PC} = 1^{--}$ may be produced. Considering the advantage of the mono-energy characteristic for the produced photon and heavy quarkonium both in the two-body final state processes, as well as the clean environment in e^+e^- collision, it seems that it is worth more efforts quantitatively to estimate the precise advantages in studying the production and the spectrum of the heavy quarkonium systems additionally with specific condition of a synthesis detectors at Z -factory. One can also conclude that it will be better if the luminosity of the Z -factory may reach to so high $\mathcal{L} \geq 10^{35 \sim 36} cm^{-2} s^{-1}$ for the heavy quarkonium studies. Moreover in Ref.[8], we report additional advantages in studying the heavy quarkonium properties at a Z -factory, but via some processes else.

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